ALGEBRA I

M. Math. I

Back Paper

Instructions: All questions carry equal marks.

1. Determine units in the ring $\mathbb{Z}/n\mathbb{Z}$.

2. Determine the image and the generators of the kernel for the ring homomorphism $\phi : \mathbb{R}[X, Y] \to \mathbb{R}[X] \times \mathbb{R}[Y]$ given by $f \mapsto (f(X, 0), f(0, Y))$.

3. Are the rings $\mathbb{Z}[X]/(X^2 + 7)$ and $\mathbb{Z}[X]/(2X^2 + 7)$ isomorphic?

4. Describe the ring obtained from $\mathbb{Z}/12\mathbb{Z}$ by adjoining the inverse of 2.

5. Let R, S be two commutative rings with unity. Determine the prime ideals in the ring $R \times S$.

6. Prove that two integer polynomials are relatively prime in $\mathbb{Q}[X]$ if and only if the ideal generated by them in $\mathbb{Z}[X]$ contains a non-zero integer.

7. Define $Z(G)$, the center of a group G. Prove that if $G/Z(G)$ is cyclic, then $G = Z(G)$.

8. Define class equation of a finite group. Give all possible class equatins of a group of order 6. Justify your answer.

9. Prove that there is no subgroup of order 15 of the permutation group S_7.

10. Define A_k, the alternating group on k symbols. Prove that any finite group is a subgroup of A_n for some n.